Algebraic approximation of analytic sets

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Abstract

In this paper we prove jet-sufficiency theorems for an equivalence relation (s-equivalence) for analytic map-germs $f$ from $\mathbb{R}^n, 0$ to $\mathbb{R}^p, 0$, and use them to prove theorems on the s-approximation of analytic sets by algebraic sets.

Two subanalytic set germs at 0 are s-equivalent if the Hausdorff distance between their intersections with the sphere centered at 0 of radius $r$ goes to zero faster than $r^s$. Then $z = j^k f(0)$ is $V_s$ sufficient (respectively $L_s$ sufficient) if, for all representatives $g$ of $z$, $V_f = f^{-1}(0)$ and $V_g$ (respectively $f(\mathbb{R}^n)$ and $g(\mathbb{R}^n)$) are s-equivalent. We prove that if the submersion points of $f$ in $V_f$ are dense in $V_f$, then some jet of $f$ is $V_s$ sufficient. We also prove that if $f^{-1}(0) = 0$, then some jet of $f$ is $L_s$ sufficient. The zero sets and image sets of such maps with s-sufficient jets are s-equivalent to algebraic sets.